USN

06MAT21

Second Semester B.E. Degree Examination, January 2013

Engineering Mathematics - II

Time: 3 hrs. Max. Marks:100

Note: 1. *Answer any FIVE full questions, choosing at least two from each part.*
2. Answer all objective type questions only in OMR sheet page 5 of the answer booklet.
3. Answer to objective type questions on sheets other than OMR will not be valued.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Choose the correct answers for the following :

i) The radius of curvature for the catenary of uniform strength $y = a \log \sec\left(\frac{x}{a}\right)$ is

A) $a \tan\left(\frac{x}{a}\right)$ B) $a \sec\left(\frac{x}{a}\right)$ C) $a \cos\left(\frac{x}{a}\right)$ D) none of these

ii) The radius of the circle of curvature is

A) 1 B) ρ C) $1/\rho$ D) ρ^2

iii) The Cauchy's mean value theorem for the function $f(x) = e^x$, $g(x) = e^{-x}$ in the interval [3, 7] is

A) C = 4 B) C = 2 C) C = 5 D) C = 7

iv) Maclaurin's expansion of e^x is

A) $1 + x + \frac{x^2}{2!} - \frac{x^3}{3!} + ...$ B) $1 + x + x^2 + x^3 + ...$

C) $1 + x - \frac{x^2}{2!} + \frac{x^3}{3!} + ...$ D) $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...$ **(04 Marks)**

b. Show that the radius of curvature of the curve $r^n = a^n \cos n\theta$ varies inversely as r^{n-1}. **(04 Marks)**

c. State and prove Cauchy's mean value theorem. **(06 Marks)**

d. Expand log (1 + sin x) in powers of x by Maclaurin's theorem upto the term containing x^4. **(06 Marks)**

2 a. Choose the correct answers for the following :

i) $\lim\limits_{x \to 0} \frac{1 - \cos x}{x \log(1 + x)}$ is equal to

A) 1 B) 2 C) 1/2 D) $\sqrt{2}$

ii) If $rt - s^2 > 0$, $r > 0$ then f(a, b) is

A) maximum value of f(x, y) B) minimum value of f(x, y)
C) saddle point D) none of these

iii) The minimum value of $x^2 + y^2 + z^2$ when $x + y + z = 3a$ is

A) 3a B) $9a^2$ C) $3a^2$ D) $3a^3$

iv) If x, y, z are the angles of a triangle, then the maximum value of cos x. cos y. cos z is

A) 5/8 B) 3/8 C) 7/8 D) 1/8 **(04 Marks)**

2 b. Evaluate $\lim_{x \to 0} \left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}}$. **(04 Marks)**

c. Expand $\tan^{-1}\left(\frac{y}{x}\right)$ about the point (1, 1) upto third degree terms. **(06 Marks)**

d. Examine the function sin x + sin y + sin (x + y) for extreme values. **(06 Marks)**

3 a. Choose the correct answers for the following :

i) $\int_0^1 \int_x^{\sqrt{x}} xy \, dydx$ is equal to

A) 1/12 B) 1/24 C) 1/48 D) 1/17

ii) $\int_{-1}^{1} \int_0^{z} \int_{x-z}^{x+z} (x + y + z) \, dydxdz$ is equal to

A) −1 B) +1 C) 0 D) 1/2

iii) The value of $\Gamma\left(\frac{1}{2}\right)$ is equals to

A) $\sqrt{\pi}$ B) π C) $\pi/2$ D) $\sqrt{\pi}/2$

iv) The value of $\beta\left(\frac{1}{2}, \frac{1}{2}\right)$ is equal to

A) 2 B) 5.678 C) 3.1416 D) 2.718 **(04 Marks)**

b. Evaluate $\iint y \, dxdy$ over the region bounded by the first quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. **(04 Marks)**

c. Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z} \, dzdydx$. **(06 Marks)**

d. Express $\int_0^1 x^m (1 - x^n)^p dx$ in terms of beta functions and hence evaluate $\int_0^1 x^5 (1 - x^3)^{10} dx$. **(06 Marks)**

4 a. Choose the correct answers for the following :

i) $\vec{F}$ is said to be irrotational if

A) $\oint_c \vec{F}.\vec{dr} = 0$ B) $\oint_c \vec{F} \times \vec{dr} = 0$ C) $\oint_c \vec{F} = 0$ D) None of these

ii) If $\vec{F} = xyi + yzj + zxk$, then $\int_c \vec{F}.\vec{dr}$ is where 'c' is the curve is given by x = t, y = t^2, z = t^3, $-1 \le t \le 1$ is

A) 7/10 B) 10/7 C) 8/7 D) 7/9

iii) Gauss divergence theorem gives the relation between

A) a surface integral and a volume integral

B) a line integral and a volume integral

C) a line integral and a surface integral

D) two volume integrals

iv) The divergence of the vector $\vec{V} = z\sin\phi\, \hat{e}_p + z\cos\phi\, \hat{e}_\phi - p\cos\phi\, \hat{e}_z$ is

A) zero B) -1 C) +1 D) 2 **(04 Marks)**

4 b. Find the area of the asteroid $x = a\cos^3\theta$, $y = a\sin^3\theta$ by employing Green's theorem. **(04 Marks)**

c. Derive the expression for curl $\vec{A}$ in orthogonal curvilinear coordinates. **(06 Marks)**

d. Express the vector $\vec{A} = zi - 2xj + yk$ in cylindrical coordinates. **(06 Marks)**

PART – B

5 a. Choose the correct answers for the following :

i) The general solution of $(D^2 + a^2)y = 0$ is

A) $y = C_1\cos ax + C_2\sin ax$ B) $y = C_1e^{ax} + C_2e^{-ax}$

C) $y = C_1\cos ax - C_2\sin ax$ D) None of these

ii) The P.I of the differential equation $6y'' + 17y' + 12y = e^{-x}$ is

A) e^{-x} B) $-e^{-x}$ C) $2e^{-x}$ D) $3e^{-x}$

iii) If $f(D) = D^2 + 5$, $\frac{1}{f(D)}\sin 2x$ is

A) 1 B) −1 C) 0 D) 2

iv) By the method of undetermined coefficients y_p of $y'' + 3y' + 2y = 12x^2$ is

A) $a + bx + cx^2$ B) $a + bx$ C) $ax + bx^2 + cx^3$ D) None of these **(04 Marks)**

b. Find the PI of $(D^3 + 1) = \cos(2x - 1)$. **(04 Marks)**

c. Solve the equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4$. **(06 Marks)**

d. Solve by the method of undetermined coefficients $\frac{d^2y}{dx^2} + \frac{2dy}{dx} + 4y = 2x^2 + 3e^{-x}$. **(06 Marks)**

6 a. Choose the correct answers for the following :

i) By the method of variation of parameters the value w is called

A) Wronskian of the function B) Euler's function

C) Leibnitz's function D) None of these

ii) The general solution of $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = 0$ is

A) $C_1x + C_2x^2$ B) $C_1x^{-1} + C_2x^{-2}$ C) $(C_1x + C_2x)e^{-x}$ D) None of these

iii) To transform $\frac{x^2d^2y}{dx^2} - \frac{xdy}{dx} + y = \log x$ into a linear differential equation with constant coefficient, put x =

A) e^{-t} B) e^t C) e^{-2t} D) log t

iv) The value of Wronskian w for the equation $y'' + 4y = 4\sec^2 2x$ is

A) 2 B) −2 C) 1 D) −1 **(04 Marks)**

b. Solve $\frac{x^2d^2y}{dx^2} + \frac{xdy}{dx} + y = \log x\sin(\log x)$. **(04 Marks)**

c. Solve by the method of undetermined coefficients $y'' - 5y' + 6y = e^{3x} + x$. **(06 Marks)**

d. Solve $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = \sin 2[\log(1+x)]$. **(06 Marks)**

7 a. Choose the correct answers for the following :

i) Laplace transform of (t cos at) is

A) $\frac{s^2-a^2}{(s^2+a^2)^2}$ B) $\frac{s^2+a^2}{(s^2-a^2)^2}$ C) $\frac{s^2}{s^2-a^2}$ D) $\frac{s^2}{(s^2-a^2)^2}$

ii) Laplace transform of cos 3t is

A) $\frac{s}{s^2+9}$ B) $\frac{s}{s^2+3}$ C) $\frac{s}{s^2-9}$ D) None of these

iii) Laplace transform of f′(t) is

A) SL{f(t)} – f(0) B) SL{f(t)} – f′(0) C) F(S) D) None of these

iv) A unit step function is defined as

A) $u(t-a)=\begin{cases}0, & t<a\\ 1, & t\geq a\end{cases}$ B) $u(t-a)=\begin{cases}1, & t>a\\ 0, & t\geq a\end{cases}$

C) $t-a=0$ D) None of these **(04 Marks)**

b. Find the Laplace transform of $e^{-4t}\ t^{-5/2}$. **(04 Marks)**

c. Find the Laplace transform of $\frac{\cos at-\cos bt}{t}$. **(06 Marks)**

d. Find the Laplace transform of the function using unit step function

$$f(t)=\begin{cases}1, & 0<t<1\\ t, & 1<t\leq 2\\ t^2, & t>2\end{cases}$$ **(06 Marks)**

8 a. Choose the correct answers for the following :

i) Inverse Laplace transform of $\frac{s}{(s^2+a^2)^2}$ is

A) $\frac{t^2\sin at}{2a}$ B) $\frac{t\sin at}{2a}$ C) $\frac{t\cos at}{2a}$ D) None of these

ii) Inverse Laplace transform of $\log\left(\frac{s+a}{s+b}\right)$ is

A) $\frac{e^{at}-e^{bt}}{t}$ B) $\frac{e^{-bt}-e^{-at}}{t}$ C) $\frac{e^{bt}-e^{at}}{t}$ D) None of these

iii) Inverse Laplace transform of $\frac{1}{s}f(s)$ is

A) $\int_0^t f(t)dt$ B) $\int_0^t t\,dt$ C) $\int_0^t tf(t)dt$ D) None of these

iv) $L^{-1}\left(\frac{1}{s^n}\right)$ is possible only when n is

A) $n>1$ B) $n\geq -1$ C) $n=1$ D) $n<1$ **(04 Marks)**

b. Find the inverse Laplace transform of $\frac{3s+2}{s^2-s-2}$. **(04 Marks)**

c. Using the convolution theorem, obtain the inverse Laplace transform of $\frac{s^2}{(s^2+a^2)^2}$. **(06 Marks)**

d. Solve the DE $y''+4y'+3y=e^{-t}$ with y(0) = 1, y′(0) = 1 using Laplace transform. **(06 Marks)**

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